NONSTEADY HEAT TRANSFER OF A MULTILAYER WALL WITH UNEQUAL CONTACT AREAS BETWEEN THE LAYERS

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The nonsteady heat-transfer problem in a system of contacting bodies with unequal contact surfaces is solved. An example of using the solution for the case of thermal tubes is given.

In modern equipment, especially in mechanical engineering and electronics, the transmission of the heat flux to the surrounding medium is often through a system of bodies which are in contact through surfaces of unequal dimensions.

In a series of cases, the thermal model of this practically important system of bodies may be written in the form of a set of contacting plates (multilayer wall) with unequal dimensions over the length and width and various contact surfaces relative to one another, the temperature of which only varies over the thickness of each layer in the direction of heat-flux motion (Fig. 1).

The difference here from the classical formulation of the problem of nonsteady heat transfer in a multilayer wall is that in the heat-transfer equations account must be taken of the difference in specific thermal fluxes transmitted to each layer. In the formulation here proposed, this difference is taken into account by introducing a dimensionless parameter in the boundary conditions; this parameter expresses the ratio of the surface area of the given layer to the surface area of the first layer encountered by the scattered power.

On the basis of the foregoing, the nonsteady heat transfer for an equivalent one-dimensional thermal model is written in the form of a system of differential heat-conduction equations for contacting layers with ideal contact at the boundaries

$$\begin{aligned} c_1\rho_1 \frac{\partial T_1}{\partial \tau} &= \lambda_1 \frac{\partial^2 T_1}{\partial x^2}, \\ c_2\rho_2 \frac{\partial T_2}{\partial \tau} &= \lambda_2 \frac{\partial^2 T_2}{\partial x^2}, \\ &\cdots\cdots \\ c_i\rho_i \frac{\partial T_i}{\partial \tau} &= \lambda_i \frac{\partial^2 T_i}{\partial x^2}, \\ &\cdots\cdots \\ c_m\rho_m \frac{\partial T_m}{\partial \tau} &= \lambda_m \frac{\partial^2 T_m}{\partial x^2}. \end{aligned} \tag{1}$$

The boundary conditions for the contacting layers are

$$x = 0 \qquad q_1 = P/S_1 = -n_1\lambda_1 \frac{\partial T_1}{\partial x};$$

$$x = \delta_1 \qquad T_1 = T_2,\ n_1\lambda_1 \frac{\partial T_1}{\partial x} = n_2\lambda_2 \frac{\partial T_2}{\partial x};$$

$$x = \delta_1 + \delta_2 \qquad T_2 = T_3,\ n_2\lambda_2 \frac{\partial T_2}{\partial x} = n_3\lambda_3 \frac{\partial T_3}{\partial x};$$

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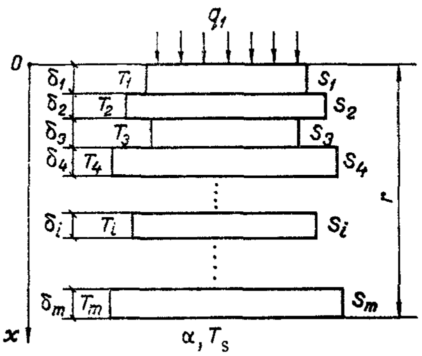

Fig. 1

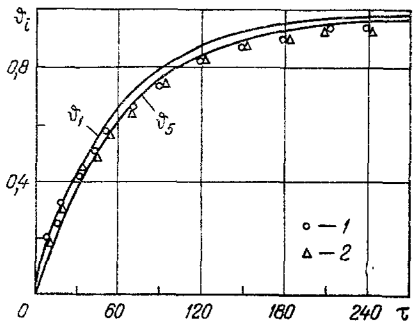

Fig. 2

Fig. 1. Theoretical thermal model of system: q_1, W/m^2; T, °K; S, m^2; δ, m; X, m; α, $W/m^2 \cdot {}^\circ K$.

Fig. 2. Results of calculating nonsteady heat transfer in TT and comparison with experimental data: the curves correspond to calculation: ($\vartheta_1 = (T_1 - T_s)/(T_{1st} - T_s)$, $\vartheta_5 = (T_5 - T_s)/(T_{5st} - T_s)$; 1) experimental points on the surface of the evaporation region; 2) at the condensation region. T, °K; τ, sec.

$$\dots\dots\dots$$

$$x = \sum_1^i \delta_i \quad T_{i-1} = T_i, \; n_{i-1}\lambda_{i-1}\frac{\partial T_{i-1}}{\partial x} = n_i\lambda_i\frac{\partial T_i}{\partial x};$$

$$\dots\dots\dots$$

$$x = \sum_1^{m-1} \delta_i \quad T_{m-1} = T_m, \; n_{m-1}\lambda_{m-1}\frac{\partial T_{m-1}}{\partial x} = n_m\lambda_m\frac{\partial T_m}{\partial x}; \tag{2}$$

$$x = \sum_1^m \delta_i = r \quad -\lambda_m\frac{\partial T_m}{\partial x} = \alpha(T_m - T_s).$$

The initial conditions for all the layers are

$$\tau = 0 \quad T_i = T_s, \quad 1 \leqslant i \leqslant m. \tag{3}$$

The dimensionless parameter $n_i = S_i/S_1$ expressing the ratio of the surface area of the i-th layer to that of the first layer, to which the specific heat flux $q_1 = P/S_1$ is directly supplied, takes account of the difference in contact areas of the individual contacting layers.

Thus, as a result of introducing the dimensionless parameter n_i in the system of equations, the mathematical description of the problem obtained reduces to the classical formulation of a multilayer wall. Its solution by the grid method is well known, but not always convenient in practice; therefore, one of the possible approximate analytical solutions based on the method of instantaneous regular conditions for a multilayer wall [1] is presented below. In this case, the heat-transfer process is divided into two periods: The inertial period and the regular period; this allows the duration of the first period to be determined analytically and the temperature field over the thickness of the multilayer wall to be calculated as a function of the time.

Assuming that

$$E_i = c_i\rho_i\delta_i n_i, \; R_i = \delta_i/\lambda_i n_i, \; R_m = 1/\alpha n_m,$$

the analytical dependences of [1] may be used to solve the problem of a multilayer wall.

The use of the method is demonstrated for the example of thermal tubes (TT) with a low axial heat conduction of the body without a transport section and constant conductivity, working in evaporative conditions.

The thermal model of TT operating in evaporative conditions usually takes the form of two-layer plates (walls of the body and core); the first is the evaporation region, to which the constant heat flux q_1 is supplied; the second is the condensation region, from which heat is extracted by the specified heat-transfer law to the surrounding medium. The thermal contact between the evaporation and condensation regions is by means of a vapor flux with a temperature that is constant over the whole length of the first channel; the heat-transfer intensity of this flux with the surface of the capillary structure tends to infinity, which allows boundary conditions of the fourth kind to be used for the surfaces of the evaporation and condensation regions [2].

Thus, the one-dimensional model of the TT may be written in the form of four isothermal layers with unequal contact surfaces: the wall of the evaporation region, the capillary structures of the evaporation and condensation regions, and the wall of the condensation region (i.e., m = 4), the temperature field in which varies only over the thickness.

In contrast to the solution considered in [2], the present version allows the length of the preregular (inertial) period of TT thermal conditions to be analytically estimated. Using this solution, the heat fluxes and temperature field over the thickness of all the layers may also be calculated as a function of the time, i.e., the time for the TT to reach a steady state may be determined.

By analogy with [1], the length of the inertial period is determined from the following expression

$$\tau_I = \frac{1}{\sum\limits_{i=1}^{4} E_i}\left[\frac{1}{2}\frac{\delta_1}{\lambda_1}E_1\left(\frac{1}{2}E_1+E_2+E_3+E_4\right)-\right.$$
$$-\frac{1}{12}\frac{\delta_1}{\lambda_1}E_1^2+\frac{\delta_2}{\lambda_2}\left(E_1+\frac{1}{2}E_2\right)\left(\frac{1}{2}E_2+E_3+E_4\right)- \tag{4}$$
$$-\frac{1}{12}\frac{\delta_2}{\lambda_2}E_2^2+\frac{\delta_3}{n\lambda_3}\left(E_1+E_2+\frac{1}{2}E_3\right)\left(\frac{1}{2}E_3+E_4\right)-$$
$$\left.-\frac{1}{12}\frac{\delta_3}{n\lambda_3}E_3^2+\frac{\delta_4}{n\lambda_4}\left(E_1+E_2+E_3+\frac{1}{2}E_4\right)\frac{1}{2}E_4-\frac{1}{12}\frac{\delta_4}{n\lambda_4}E_4^2\right],$$

where

$$E_1=c_1\rho_1\delta_1;\; E_2=c_2\rho_2\delta_2;\; E_3=c_3\rho_3\delta_3 n;\; E_4=c_4\rho_4\delta_4 n;$$
$$n_1=S_1/S_1=1;\; n_2=S_2/S_1=1\;(S_2=S_1);\; n_3=S_3/S_1=n_4=S_4/S_1=n.$$

For TT, $n = S_c/S_e$ is the ratio of surface areas of the condensation and evaporation regions.

The time for the regular period of TT thermal conditions to begin is determined directly from Eq. (4).

In the regular period, the heat fluxes at the layer boundaries (q_5, q_4, q_3, q_2) and temperatures (T_5, T_4, T_3, T_2, T_1) are calculated as a function of the time from the analytical dependences

$$q_5=q_1\left[1-\exp\left(-\frac{\tau-\tau_I}{\tau_a-\tau_I}\right)\right],$$
$$q_4=q_1-\frac{E_1+E_2+E_3}{\sum\limits_{i=1}^{4}E_i}(q_1-q_5), \tag{5}$$
$$q_3=q_1-\frac{E_1+E_2}{\sum\limits_{i=1}^{4}E_i}(q_1-q_5),\qquad q_2=q_1-\frac{E_1}{\sum\limits_{i=1}^{4}E_i}(q_1-q_5);$$

$$T_5 = T_s + \frac{q_1}{n\alpha}\left[1 - \exp\left(-\frac{\tau - \tau_I}{\tau_a - \tau_I}\right)\right],$$

$$T_4 = T_5 + \frac{(q_4 + q_5)\,\delta_4}{2n\lambda_4},$$

$$T_3 = T_4 + \frac{(q_3 + q_4)\,\delta_3}{2n\lambda_3}, \qquad (6)$$

$$T_2 = T_3 + \frac{(q_2 + q_3)\,\delta_2}{2\lambda_2},$$

$$T_1 = T_2 + \frac{(q_1 + q_2)}{2\lambda_1}.$$

For simplicity, expressions are given here only for calculating the heat fluxes and temperatures at the layer boundaries.

The accumulation time τ_a is determined from the formula

$$\tau_a = E_1\left(\frac{1}{n\alpha} + \frac{\delta_4}{\lambda_4 n} + \frac{\delta_3}{\lambda_3 n} + \frac{\delta_2}{\lambda_2} + \frac{1}{2}\,\frac{\delta_1}{\lambda_1}\right) + E_2\left(\frac{1}{n\alpha} + \frac{\delta_4}{\lambda_4 n} + \frac{\delta_3}{\lambda_3 n} + \frac{1}{2}\,\frac{\delta_2}{\lambda_2}\right) + E_3\left(\frac{1}{n\alpha} + \frac{\delta_4}{\lambda_4 n} + \frac{1}{2}\,\frac{\delta_3}{\lambda_3 n}\right) + E_4\left(\frac{1}{n\alpha} + \frac{1}{2}\,\frac{\delta_4}{\lambda_4 n}\right). \qquad (7)$$

The heating process ends when a steady state sets in; in this steady state, the amount of heat supplied to the evaporation region is equal to that removed from the condensation region by the surrounding medium.

For the steady state

$$T_{1\,st} - T_s = q_1\left(\frac{\delta_1}{\lambda_1} + \frac{\delta_2}{\lambda_2} + \frac{\delta_3}{\lambda_3 n} + \frac{\delta_4}{\lambda_4 n} + \frac{1}{n\alpha}\right), \qquad (8)$$

$$T_{5st} - T_s = q_1/n\alpha.$$

The order of calculating the TT heating to the steady state is as follows. From Eqs. (4) and (7), τ_I and τ_a are determined. If $\tau_a \gg \tau_I$, the heat fluxes at the layer boundaries q_5, q_4, q_3, $q_2 = f(\tau)$ and the temperatures T_5, T_4, T_3, T_2, and $T_1 = f(\tau)$ are calculated from Eqs. (5) and (6) at the initial temperature $T_i = T_s$. If not, by analogy with [1], it is necessary to calculate the heat fluxes and temperatures for the layers at the end of the inertial period. As a result, the complete time picture of the transition of all the TT elements to steady thermal conditions is obtained. It may now be shown how the heating times of individual TT elements are calculated for the following conditions: the frame and capillary structure are made from stainless steel and the heat carrier is water; the scattered power is 15 W.

The surface area of the evaporation region $S_e = S_1 = 1.29 \cdot 10^{-3}$ m^2; that of the condensation region is $S_c = S_5 = 5.42 \cdot 10^{-3}$ m^2 (hence, $q_1 = P/S_1 = 11{,}670$ W/m^2; $n = S_c = S_e = 4.2$). Heat transfer with the surrounding medium in the condensation region is by induced convection ($\alpha = 80$ W/m$^2 \cdot$K); the temperature of the surrounding medium $T_s = 293$°K.

The thermophysical and geometric parameters of the layers are as follows: $c_1 = c_4 = c_e = c_c = 502$ J/kg·°K; $c_2 = c_3 = c_e^c = c_c^c = 1463$ J/kg·°K; $\rho_1 = \rho_4 = 7800$ kg/m^3; $\rho_2 = \rho_3 = 6000$ kg/m^3; $\delta_1 = \delta_4 = 0.5 \cdot 10^{-3}$ m; $\delta_2 = \delta_3 = 0.22 \cdot 10^{-3}$ m; $\lambda_1 = \lambda_4 = 13$ V/m·°K; $\lambda_2 = \lambda_3 = 1.07$ W/m·°K. Hence $E_1 = c_1\rho_1\delta_1 = 1970$ J/m^2·°K; $E_2 = c_2\rho_2\delta_2 = 1930$ J/m^2·°K; $E_3 = c_3\rho_3\delta_3 n = 7900$ J/m^2·°K; $E_4 = c_4\rho_4\delta_4 n = 8270$ J/m^2·°K.

Substituting these values into Eqs. (4) and (7), it is found that $\tau_I = 0.8$ sec, $\tau_a = 61.4$ sec. Then q_5, q_4, q_3, q_2 and T_5, T_4, T_3, T_2, $T_1 = f(\tau)$ are determined from Eqs. (5) and (6). The results of the calculations are shown in Fig. 2. For clarity, the time dependence of the TT heating is shown in dimensionless form: $\vartheta_i = (T_i - T_s)/(T_{ist} - T_s) = f(\tau)$ for the wall of the evaporation region ϑ_1 ($T_i = T_1$) and the wall of the condensation region ϑ_5 ($T_i = T_5$); T_{1st} and T_{5st} are determined from Eq. (8).

The numerical and analytical calculations practically coincide, and therefore ϑ_1 and ϑ_5 are shown by single curves. The inertial time obtained numerically (τ_I = 0.5 sec when ϑ_1 = 0.001) is also very small for the specified initial data, which confirms the hypothesis that, in such types of TT, thermal conditions set in practically instantaneously.

NOTATION

T, T_S, temperature of layer and surrounding medium; c, λ, ρ, δ, S, specific heat, heat conduction, density, thickness, and surface area of the given layer; n, dimensionless parameter expressing the ratio of the surface area of the given layer to that of the first layer; τ, time; r, total thickness of all the layers; q, specific heat flux; R, thermal resistance; α, heat-transfer coefficient; P, scattered power; i, layer number; m, total number of layers; e, evaporation region; c, condensation region; C, capillary structure.

LITERATURE CITED

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